

Because $d y / d x$ is a function of $t$, you can use Theorem 9.2 repeatedly to find higher-order derivatives.

## HIGHER ORDER DERIVATIVES PARAMETRICALLY

$$
\begin{array}{ll}
\frac{d^{2} y}{d x^{2}}=\frac{d}{d x}\left[\frac{d y}{d x}\right]=\frac{\frac{d}{d t}\left[\frac{d y}{d x}\right]}{d x / d t} & \text { Second derivative } \\
\frac{d^{3} y}{d x^{3}}=\frac{d}{d x}\left[\frac{d^{2} y}{d x^{2}}\right]=\frac{\frac{d}{d t}\left[\frac{d^{2} y}{d x^{2}}\right]}{d x / d t} & \text { Third derivative }
\end{array}
$$

Example 1: Finding the Second Derivative of a Parametric Equation of the following:
a.) $x(t)=e^{t}, y(t)=t e^{-t}$
b.) $x(t)=\cos t, y(t)=\sin 2 t, 0<t<\pi$

## Example 2: Finding Slope and Concavity

For the curve given by $x=\sqrt{t}$ and $y=\frac{1}{4}\left(t^{2}-4\right), t \geq 0$, find the slope and concavity at the point $(2,3)$.


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## Fundamental Theorem of Calculus with Parametric Equations

Example 3: Position Desired
A particle moving along a curve in the $x y$-plane is at position $(x(t), y(t))$ at time $t$, where $\frac{d x}{d t}=\ln (t+1), \frac{d y}{d t}=\arcsin \left(e^{-t^{2}}\right)$ for $t \geq 0$. At time $t=1$ the particle is at position $(2,5)$.
b.) Find the $x$-coordinate of the position of the particle at time $t=3$.

