

With a slight modification, we can change the concept of finding the area of a region <u>under</u> a curve (that lies above the *x*-axis) to finding the area of a region <u>between</u> two curves.

Consider the following graphs of y = f(x) and y = g(x) that are continuous on the interval [2,4].



# Area of a Region Between Two Curves

If f and g are continuous on [a, b] and  $g(x) \le f(x)$  for all x in [a, b], then the area of the region bounded by the graphs of f and g and the vertical lines x = a and x = b is

$$A = \int_{a}^{b} [f(x) - g(x)] \, dx$$

## **Example 1**: Finding the Area of a Region Between Curves.

Find the area of the region bounded by the graphs of  $y = x^2 + 2$ , y = -x, x = 0 and x = 1. Sketch the graph and shade the region.



**Example 2**: A Region Lying Between Two Intersecting Graphs. Find the area of the region bounded by the graphs of  $f(x) = 2 - x^2$  and g(x) = x. Sketch the graph and shade the region.



СНА			
4	Topic: 8.5	Finding Areas Between Curves Expressed as Functions of y	
Learning Objective CHA-5.A: Calculate areas in the plane using the definite integral.			

**Example 3:** Horizontal Representative Rectangles. Find the area of the region bounded by the graphs of  $x = 3 - y^2$  and x = y + 1. Sketch the graph and shade the region.



**Example 4:** Find the area bound by the curves  $x = y^2 - 2$ , y = -1, y = 1, and  $x = e^y$  as shown in the graph. Shade the region that is bounded by those given equations.



СНА			
2	Topic: 8.6	Finding Areas Between Curves that Intersect at More than Two Points	
Learning Objective CHA-5.A: Calculate areas in the plane using the definite integral.			

**Example 5:** Curves That Intersect at More Than Two Points. Find the area of the regions bounded by the graphs of  $f(x) = 3x^3 - x^2 - 10x$  and  $g(x) = -x^2 + 2x$ . Sketch the graph and shade the region.

