## CHA

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Topic: 8.4
Finding Areas Between Curves Expressed as Functions of $\boldsymbol{x}$

Learning Objective CHA-5.A: Calculate areas in the plane using the definite integral.

With a slight modification, we can change the concept of finding the area of a region under a curve (that lies above the $x$ axis) to finding the area of a region between two curves.

Consider the following graphs of $y=f(x)$ and $y=g(x)$ that are continuous on the interval $[2,4]$.



Area of region
between $f$ and g
$=$


Area of region under g
$\int_{1}^{3}[f(x)-g(x)] d x \quad \int_{1}^{3} f(x) d x \quad-\quad \int_{1}^{3} g(x) d x$

## Area of a Region Between Two Curves

If $f$ and $g$ are continuous on $[a, b]$ and $g(x) \leq f(x)$ for all $x$ in $[a, b]$, then the area of the region bounded by the graphs of $f$ and $g$ and the vertical lines $x=a$ and $x=b$ is

$$
A=\int_{a}^{b}[f(x)-g(x)] d x
$$

## Example 1: Finding the Area of a Region Between Curves.

Find the area of the region bounded by the graphs of $y=x^{2}+2, y=-x, x=0$ and $x=1$. Sketch the graph and shade the region.


## Example 2: A Region Lying Between Two Intersecting Graphs.

Find the area of the region bounded by the graphs of $f(x)=2-x^{2}$ and $g(x)=x$. Sketch the graph and shade the region.


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## Example 3: Horizontal Representative Rectangles.

Find the area of the region bounded by the graphs of $x=3-y^{2}$ and $x=y+1$. Sketch the graph and shade the region.


Example 4: Find the area bound by the curves $x=y^{2}-2, y=-1, y=1$, and $x=e^{y}$ as shown in the graph. Shade the region that is bounded by those given equations.


## CHA

| 2 | Topic: 8.6 |
| :--- | :--- | Finding Areas Between Curves that Intersect at More than Two Points

Learning Objective CHA-5.A: Calculate areas in the plane using the definite integral.

Example 5: Curves That Intersect at More Than Two Points.
Find the area of the regions bounded by the graphs of $f(x)=3 x^{3}-x^{2}-10 x$ and $g(x)=-x^{2}+2 x$.
Sketch the graph and shade the region.


