

Using Fundamental Identities

Notes: 5.1

Fundamental Trigonometric Identities

Reciprocal Identities:

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient Identities:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

Cofunction Identities:

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\left(\frac{\pi}{2} = 90^\circ\right)$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Negative Angle Identities:

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

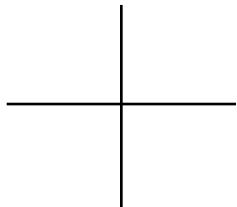
$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

Example 1: a. Use the values $\sin u = \frac{1}{3}$ and $\cos u < 0$ to find the values of all six trigonometric functions.



$$\sin u =$$

$$\cos u =$$

$$\tan u =$$

$$\csc u =$$

$$\sec u =$$

$$\cot u =$$

b. Use a Pythagorean Identity to find $\cos u$.

Simplifying a Trigonometric Expression

Example 2:

$$\sin \theta \csc \theta$$

$$\cot \alpha \sin^2 \alpha$$

$$\sec \beta (\sec \beta - \cos \beta)$$

Factoring Methods from Algebra

Difference of Squares: $1 - a^2 =$

Example 3:

Factor: $1 - \cos^2 x =$

Factoring a Quadratic Trinomial $a \neq 1$: $2a^2 - 7a + 6 =$

Example 4: Factor: $2 \csc^2 x - 7 \csc x + 6 =$

Factoring a Quadratic Trinomial $a = 1$: $a^2 + 3a + 2 =$

Example 5: Factor: $\sec^2 x + 3\tan x + 1$

Example 6: Simplify $\csc t - \cos t \cot t$

Example 7: Simplify $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

Example 8: Simplify $\frac{\cos^2 y}{1-\sin y}$

Example 9: Simplify: $(\cot x + \csc x)(\cot x - \csc x)$

Example 10: Simplify: $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x}$