

**The number  $e$** 

$$e \approx 2.71828$$

Example:

Find:  $e^{2.5}$

(use the  $e^x$  key on your calculator)**Natural Logarithm – a logarithm with base  $e$** 

$$\log_e x \rightarrow \ln x$$

Examples:

$$\ln 1.75 \approx 0.5596$$

$$\ln \frac{2}{3} \approx -0.4055$$

$$\ln e = 1$$

$$\ln e^5 = 5$$

**Simplifying Natural Logarithms.**

Write each expression as a single natural logarithm.

Example:  $5 \ln 2 - \ln 4$

$$\ln 2^5 - \ln 4$$

$$\ln \frac{2^5}{4}$$

$$\ln \frac{32}{4} = \ln 8$$

Example:  $2 \ln 12 + \ln 9$

$$\ln 12^2 + \ln 9$$

$$\ln 144 + \ln 9$$

$$\ln 144 \cdot 9 = \ln 1296$$

Example:  $\frac{1}{4} \ln 3 + \frac{1}{4} \ln x$

$$\ln 3^{\frac{1}{4}} + \ln x^{\frac{1}{4}}$$

$$\ln(3^{\frac{1}{4}} \cdot x^{\frac{1}{4}})$$

$$\ln(\sqrt[4]{3} \cdot \sqrt[4]{x}) = \ln \sqrt[4]{3x}$$

Example:  $3 \ln 2 + 4 \ln 3$

**APPLICATION:** The formula for a rocket's maximum velocity  $V$  (in km/sec)

$$V = -0.0098t + c \cdot \ln R$$

$t = \text{time}$

$c = \text{velocity of exhaust}$

$R = \text{ratio of } \frac{\text{mass of rocket}}{\text{mass without fuel}}$

$$t = 50$$

Find  $V$

$$V = -0.0095(50) + 2.3 \ln 22$$

$$c = 2.3$$

$$V \approx 6.6 \text{ km per second}$$

$$R = 22$$

## Solving a Natural Logarithmic Equation

$$\ln x = y \rightarrow e^y = x$$

Solve:  $\ln\left(\frac{x+2}{3}\right) = 12$

$$\log_e\left(\frac{x+2}{3}\right) = 12$$

$$e^{12} = \frac{x+2}{3}$$

$$3 \cdot e^{12} = x + 2$$

$$3 \cdot e^{12} - 2 = x$$

$$x \approx 488262.374$$

Solve:  $\ln(2x - 4)^3 = 6$

$$3 \cdot \ln(2x - 4) = 6$$

$$\ln(2x - 4) = 2$$

$$\log_e(2x - 4) = 2$$

$$e^2 = 2x - 4$$

$$e^2 + 4 = 2x$$

$$\frac{e^2 + 4}{2} = x$$

$$x \approx 5.6945$$

Solve:  $\ln(x + 4) = 3$

**Mental Math (using properties of logarithms)**

$$\ln e^3 =$$

$$\ln e =$$

$$5 \ln e =$$

$$\ln e^{4x} =$$

**Solving an Exponential Equation**

Solve:  $e^{x+1} = 30$

$$\ln e^{x+1} = \ln 30$$

$$x + 1 = \ln 30$$

$$x = \ln(30) - 1$$

$$x \approx 2.4012$$

**Change to log form OR  
take the ln of each side of equation  
remember:  $\ln e^x = x$**

Solve:  $4e^{3x} + 1.2 = 14$

$$4e^{3x} = 12.8$$

$$e^{3x} = 3.2$$

$$\ln e^{3x} = \ln 3.2$$

$$3x = \ln 3.2$$

$$x = \frac{\ln 3.2}{3}$$

$$x \approx 0.3877$$

Solve:  $e^{x-3} + 4 = 6$